Relativity

1. Two neutrons, A and B, are approaching each other along a common straight line. Each has a constant speed βc as measured in the laboratory. Show that the total energy of neutron B, as observed in the rest frame of neutron A, is:

   $\frac{(1+\beta^2)}{(1-\beta^2)} m_0 c^2$, where m_0 is the neutron rest mass.

2. A particle of rest mass m_0 and kinetic energy $2m_0c^2$ collides and stick to a stationary particle of rest mass $2m_0$. Find the rest mass M_0 and speed v of the composite particle.

   (answers: $M_0 = \sqrt{17} m_0$, $v = \frac{2\sqrt{2}c}{5}$*)*

3. You are accelerating protons in a Tandem linear electrostatic particle accelerator.
   a) Using both classical and relativistic dynamics, calculate the speed of protons accelerated to an energy of 8 MeV.
   b) By how much do both results disagree?
   c) Above which energy are the relativistic corrections larger than 1% ?
   d) Above which energy are the relativistic corrections larger than 1%, for electrons?
   e) If the maximum electrostatic potential at the center of the Tandem accelerator is 6 MV, what are the maximum relativistic corrections to the classical speed of protons.

Some useful equations:

Electrostatics

$\Delta U = q\Delta V$

$\vec{E} = \frac{\vec{F}}{q_0}$

Mechanics

$F_c = \frac{mv^2}{r}$

$K = \frac{mv^2}{2}$

Relativity:

$\beta = v/c$

$\gamma = (1-\beta^2)^{-1/2}$

Lorentz transformations

$x' = \gamma(x - vt)$

$y' = y$

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$x = \gamma\left(x' + vt'\right)$

$y = y'$

$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

Space-time invariant

$S^2 = (ct)^2 - (x)^2 = (ct')^2 - (x')^2$

Space contraction:

$L = L_0/\gamma$

Time dilation

$T = \gamma T_0$

Doppler effect

$f' = \left(\frac{1-\beta}{1+\beta}\right)^{1/2} f$

Velocity transformations

$u_x = \frac{u'_x + v}{1 + vu'_x/c^2}$

$u_y = \frac{u'_y/\gamma}{1 + vu'_x/c^2}$

$u'_x = \frac{u_x - v}{1 - vu_x/c^2}$

$u'_y = \frac{u_y/\gamma}{1 - vu_x/c^2}$

Momentum and energy

$\vec{p} = \gamma m\vec{v}$

$K = (\gamma - 1)mc^2$

$E = K + mc^2$

$E^2 = (cp)^2 + (mc^2)^2$

Physical constants

$c = 2.997 \times 10^8$ m/s

$\mu_0 = 1.26 \times 10^{-6}$ T m A^{-1}

$\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2N^{-1}m^{-2}$

$k = 8.99 \times 10^9$ Nm^2C^{-2}

$m_e = 9.11 \times 10^{-31}$ kg

$m_p = m_n = 1.67 \times 10^{-27}$ kg

$e = 1.6 \times 10^{-19}$ C

eV = 1.6×10^{-19} J

1 T = 10^4 G