

Gedanken Experiment

Twin paradox

1. Objective

- To resolve the so-called *twin paradox* (or clock paradox), one of the most famous and controversial of special relativity.

2. Theory

- The method you will use to study this paradox, is based on the relativistic Doppler effect, which states:

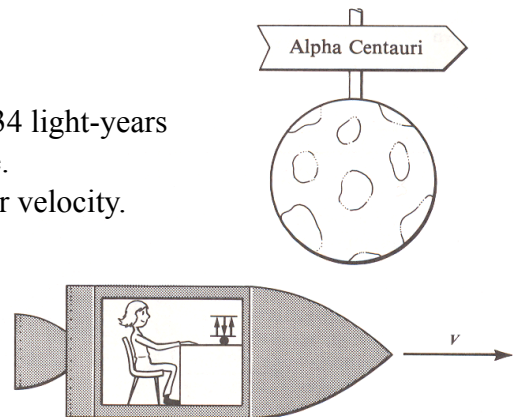
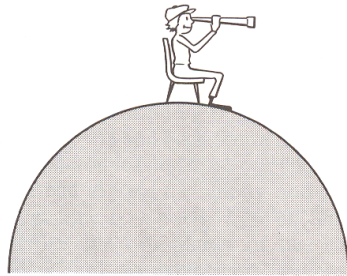
$$v' = v \left(\frac{1 - \beta}{1 + \beta} \right)^{1/2}$$

where v' is the frequency measured in S' , of a signal of frequency v in S , when S' is moving away from S with a relative velocity β .

3. The journey

The traveler's journey is the following:

- The traveller, blasts off from Earth towards Alpha-Centauri, 4.34 light-years away, and reaches a constant velocity in a negligibly short time.
- After journeying for a while, the traveller *suddenly* reverses her velocity.
- The traveller arrives back at his starting point and stops in a negligibly short time.



Each person (the traveller (**A**) and the stay-at-home (**B**)) sends equally spaced

time signals (of his/her own proper time) to the other. This means that the emission frequency was agreed upon before the journey, so both **A** and **B** emit the radio signal at the same frequency as measured from their own frame of reference.

Each observer receives as many signals as the other sends between start and finish of the trip. The cumulative count of time signals for the whole trip are then compared.

4. Calculations

1. Suppose each person is transmitting one radio pulse every 0.01 year ($= f$). Calculate the reduced rate (f') at which each observer receives the signal sent by the other, as **A** travels away from **B**.
2. Calculate the enhanced rate (f'') with which each observer will receive the signal sent as **A** travels toward **B**.
3. Will **A** and **B** start to observe the enhanced rate at the same time?
4. Fill the following table, your answers should be in terms of v , the velocity of **A**.

item	measured by B (stay-at-home)	measured by A (traveller)
Time of total trip		
Total number of signals sent		
Time of detecting A's turnaround		
Number of signals received at f'		
Time for rest of trip		
Number of signals received at f''		
Total number of signals received		
Conclusion as to the other's measure of the time taken		

5. Graph

Using the results from the previous table, plot a graph of the age difference between **A** and **B** as a function of v . (you may want to use a semi-log scale if this difference spans many orders of magnitude)