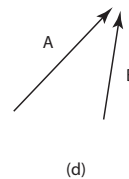
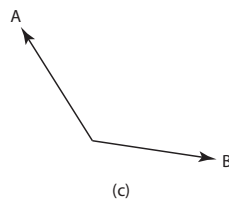
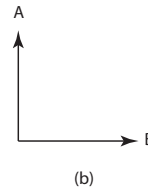
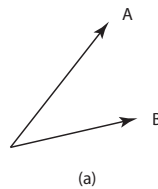


## *Problem Session*

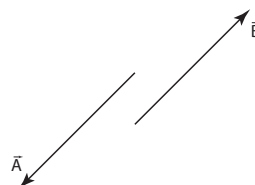
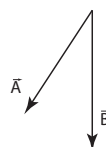
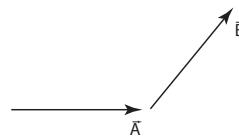
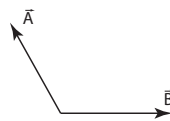
### *Vector Algebra*

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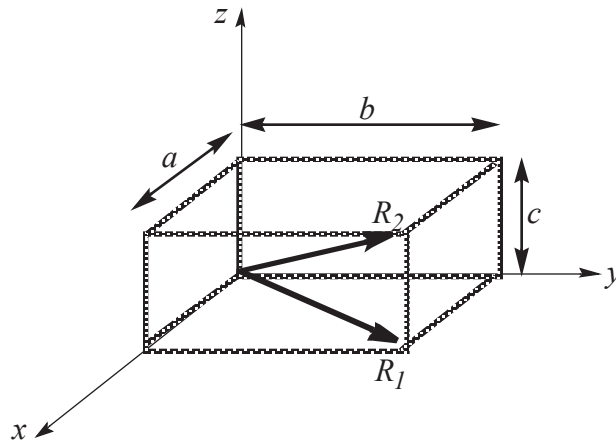
- What is the scalar product between the following vectors:
  - $\vec{A} = 3\hat{i} - 2\hat{j}$  and,  $\vec{B} = 4\hat{i} - 4\hat{j}$
  - $\vec{A} = -2\hat{i} + 4\hat{j}$  and,  $\vec{B} = 3\hat{i} - 4\hat{j}$
  - $\vec{A} = \hat{i} - 2\hat{j} + 2\hat{k}$  and,  $\vec{B} = 3\hat{j} + 4\hat{k}$
- Using the definition of the scalar product, find the angles between:
  - $\vec{A} = 3\hat{i} - 2\hat{j}$  and,  $\vec{B} = 4\hat{i} - 4\hat{j}$
  - $\vec{A} = -2\hat{i} + 4\hat{j}$  and,  $\vec{B} = 3\hat{i} - 4\hat{j}$
  - $\vec{A} = \hat{i} - 2\hat{j} + 2\hat{k}$  and,  $\vec{B} = 3\hat{j} + 4\hat{k}$
- For each pair of vectors, is the sign of the scalar product, positive (+), negative (-), or zero (0)?



- For each pair of vectors  $\vec{A}$  and  $\vec{B}$  below, determine if  $\vec{A} \times \vec{B}$  points into the page, out of the page, or is zero.



5. A rectangular parallelepiped has dimensions  $a$ ,  $b$ , and  $c$ :



- a) Obtain a vector expression for the face diagonal vector  $R_1$ . What is the magnitude of this vector?
  - b) Obtain a vector expression for the body diagonal vector  $R_2$ .
  - c) Prove that the magnitude of  $R_2$  is  $\sqrt{a^2 + b^2 + c^2}$ .
6. Use the scalar product to prove the law of cosines for a triangle:  $c^2 = a^2 + b^2 - 2ab\cos\theta$ , where  $a$ ,  $b$ , and  $c$  are the lengths of the sides of a triangle and  $\theta$  is the angle opposite side  $c$ .  
**Hint:**  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$  form three sides of an arbitrary triangle, and can be seen as three vectors having the following relationship:  $\vec{a} - \vec{b} = \vec{c}$ . Then use the fact that  $\vec{c} \cdot \vec{c} = |\vec{c}|^2$  ...
7. Using the three arbitrary vectors  $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ ,  $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ , and  $\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$ , show that the scalar product of two vectors is distributive:  $\vec{A} \cdot (\vec{B} + \vec{C}) = (\vec{A} \cdot \vec{B}) + (\vec{A} \cdot \vec{C})$ .
8. Given  $\vec{M} = 6\hat{i} + 2\hat{j} - \hat{k}$ , and  $\vec{N} = 2\hat{i} - \hat{j} - 3\hat{k}$ , calculate the vector product  $\vec{M} \times \vec{N}$ .
9. The vectors 42 cm at  $15^\circ$  and 23 cm at  $65^\circ$ , both start from the origin. Both angles are measured counterclockwise from the  $x$ -axis. The vectors form two-sides of a parallelogram.
- a) find the area of the parallelogram.
  - b) find the length of its longer diagonal.
10. A student claims that she has found a vector  $\vec{A}$ , such that :  $(2\hat{i} - 3\hat{j} + 4\hat{k}) \times \vec{A} = (4\hat{i} + 3\hat{j} - \hat{k})$ . Do you believe her? Explain.
11. If  $|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$ , what is the angle between  $\vec{A}$  and  $\vec{B}$ .

## Answers

1. a) 20, b) -22, c) 2

2. a)  $11.31^\circ$ , b)  $169.7^\circ$ , c)  $82.3^\circ$

3. a) +, b) zero, c) -, d) +

4. In the same order as the previous number: a) in, b) out, c) out, d) zero

5. a)  $\vec{R}_1 = a\hat{i} + b\hat{j}$ ,  $R_1 = \sqrt{a^2 + b^2}$

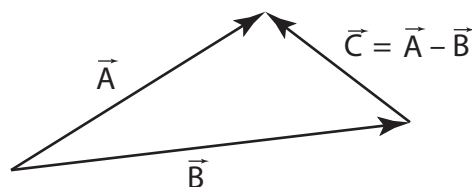
b)  $\vec{R}_2 = a\hat{i} + b\hat{j} + c\hat{k}$ , and to find  $R_2$ , you can use the fact that the diagonal  $R_1$  and the vertical side of the parallelepiped form a right angle triangle, on which we can use Pythagoras theorem to find:  $R_2 = \sqrt{a^2 + b^2 + c^2}$ .

6.  $\vec{C} = \vec{A} - \vec{B}$

$$\vec{C} \cdot \vec{C} = c^2 = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})$$

$$c^2 = \vec{A} \cdot \vec{A} + \vec{B} \cdot \vec{B} - 2\vec{A} \cdot \vec{B}$$

$$c^2 = a^2 + b^2 - 2ab \cos \theta$$



7. Calculate each side explicitly.

8. Use the equation  $\vec{A} \times \vec{B} = (A_y B_z - B_y A_z)\hat{i} + (A_z B_x - B_z A_x)\hat{j} + (A_x B_y - B_x A_y)\hat{k}$

9. If one sees the two sides of the parallelogram as two vectors  $\vec{A}$  and  $\vec{B}$ , then:

a) The area of the parallelogram is:

$$42 \times 23 \sin 50^\circ \text{ cm}^2,$$

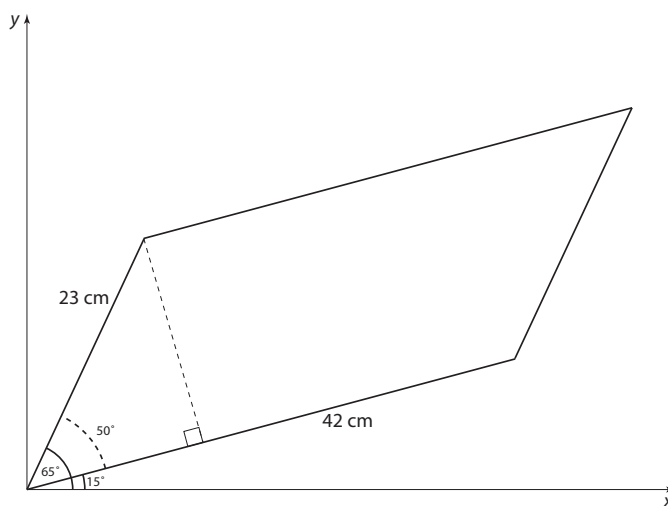
which is the same as

$$|\vec{A} \times \vec{B}| = 740 \text{ cm}^2.$$

b) The length of the longer diagonal is the magnitude of  $\vec{A} + \vec{B}$ .

write the two vectors in component form, add them, and finally, find the magnitude of the resultant vector.

$$|\vec{A} + \vec{B}| = 59.4 \text{ cm}$$



10. The resultant vector of a cross product is perpendicular to both original vectors.

The vector  $4\hat{i} + 3\hat{j} - \hat{k}$  should be perpendicular to  $2\hat{i} - 3\hat{j} + 4\hat{k}$ . Verify they are not perpendicular using the scalar product; it is therefore not possible to find such a vector  $\vec{A}$ .

11.  $ab \sin \theta = ab \cos \theta \rightarrow \theta = 45^\circ$