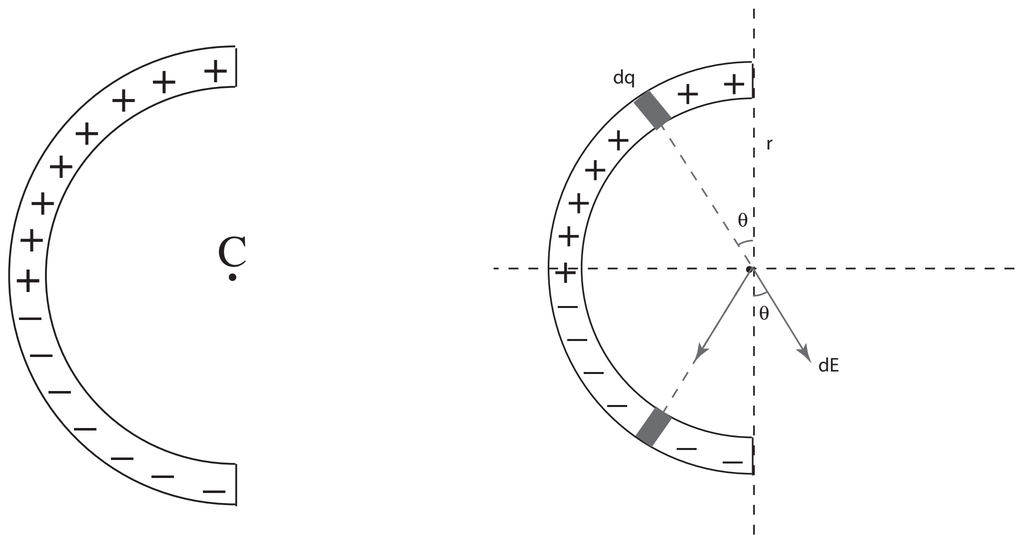
Problem Session
Electric Field

3. A thin glass rod is bent into a semicircle of radius R = 60 cm. A charge Q = +8 μC is uniformly distributed along the upper half and a charge Q = –8 μC is uniformly distributed along the lower half. Find the electric field at the center of the semicircle.



As seen in class for an element of charge dq in the top part of the distribution, the field dE points as show in the above figure. By symmetry a corresponding element in the bottom part of the distribution, will create a field such that the horizontal component of the field cancels out and the net field will be pointing downward. We are only interested in calculating the y-component of the field.

$$dE_y = dE\cos\theta = \frac{kdq}{r^2}\cos\theta$$

and since $dq = \lambda r d\theta$,

$$dE_y = \frac{k\lambda}{r}\cos\theta d\theta$$

Lambda is however not constant along the whole distribution and we must split the integral into two parts.

$$\lambda_{top} = \frac{2Q}{\pi r}$$

$$\lambda_{bottom} = \frac{-2Q}{\pi r}$$, where Q = 8 μC.

The magnitude of the total field will therefore be:

$$E = \int_0^{\pi/2} \frac{k\lambda_{top}}{r} \cos\theta d\theta + \int_{\pi/2}^{\pi} \frac{k\lambda_{bot}}{r} \cos\theta d\theta$$

$$E = \int_0^{\pi/2} \frac{k}{r}\left(\frac{2Q}{\pi r}\right) \cos\theta d\theta + \int_{\pi/2}^{\pi} \frac{k}{r}\left(\frac{-2Q}{\pi r}\right) \cos\theta d\theta$$

$$E = \frac{2Q}{\pi r^2}\left(\int_0^{\pi/2} \cos\theta d\theta - \int_{\pi/2}^{\pi} \cos\theta d\theta\right)$$

$$E = \frac{2Q}{\pi r^2}\left(\sin\theta\Big|_0^{\pi/2} - \sin\theta\Big|_{\pi/2}^{\pi}\right)$$

$$E = \frac{2Q}{\pi r^2}\left((1-0)-(0-1)\right) = \frac{4kQ}{\pi r^2}$$

So the electric field is $\vec{E} = -\frac{4kQ}{\pi r^2}\hat{j}$ or $\vec{E}$ = - 2.54 × 10^5 N/C $\hat{j}$.